Acta Technica *63* No. 1B/2018, 1–8

Development of photometric system transformation

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Abstract. The article is devoted to the determination of transition method between the photometric systems Aα, Bβ, Cγ, based on their combination by the means of rotations in the rectangular coordinate system and the subsequent interpolation of luminous intensity values in the given units. This allows us to determine all values of meridional angles in the entire range of values correctly: [-π, π] for A, B, and [0, 2π] for C. It is shown that in order to solve the problem of a regular angle grid obtaining, one can use the interpolation of scattered data on the basis of the Delaunay triangulation.

Key words. Lighting device, photometry systems, light intensity curve, goniophotometry, photometric body, triangulation grid, the interpolation of scattered data..

1. Introduction

The spatial-angular distribution of the light intensity is determined during goniophotometric measurements and can be specified in one of the three photometric systems Aα, Bβ, Cγ (type A, type B, type C) [1–4]. The choice of a particular system for a particular type of light source (LS) or illuminating devices (ID) is not regulated strictly by standards, although there are some recommendations. For example, spotlights are more suitable for photo-metering in the Bβ system [1–3], car headlights - in Aα system [3], and office and street lamps - in Cγ system. If the kinematic scheme of a goniophotometer involves the rotation of a LS, and a discharge lamp is measured, then the system must be chosen in which its operation position does not change. Often, the choice can be dictated just by the convenience of measuring. There can be situations in lighting engineering practice, when photo metering is carried out in one system, and the results must be presented in another

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system. For example, it is necessary to compare the results of two goniophotometers measurements whose kinematic schemes implement different systems of photometry. References [1, 3] contain the formulas for the transition between photometry systems. However, as will be shown below, this issue has not been studied fully.

2. Method

Photometric systems are spherical coordinate systems that are oriented in a certain way relative to photometric, longitudinal and transverse axes of a LD [1]. The combination of photo-metering systems can be carried out either by the use of the basic concepts and rules of spherical geometry [5], or via matrix transformations in the Cartesian coordinate system. Both methods lead to the same results, but in this paper we prefer the latter one because of its more economical, easy-to-remember form of record. As is well known, the transition from spherical coordinates to rectangular ones is carried out according to the following formulas:

$$x = r \sin\theta \cos\varphi \,,$$

$$x = r \sin\theta \sin\varphi \,,$$

$$z = r \cos\theta \,. \tag{1}$$

Now, let us explain the way of transition to rectangular coordinates associated with the photometric systems Cγ, Bβ, Aα. The coordinate axes in all three systems are the transverse, longitudinal and photometric axes of an LD. The positive directions of the coordinate axes in the systems Cγ, Bβ, Aα determine the triples of the unit vectors ($\boldsymbol{i}_{\mathrm{C}}$, $\boldsymbol{j}_{\mathrm{C}}$, $\boldsymbol{k}_{\mathrm{C}}$), ($\boldsymbol{i}_{\mathrm{B}}$, $\boldsymbol{j}_{\mathrm{B}}$, $\boldsymbol{k}_{\mathrm{B}}$) and ($\boldsymbol{i}_{\mathrm{A}}$, $\boldsymbol{j}_{\mathrm{A}}$, $\boldsymbol{k}_{\mathrm{A}}$), respectively (see Figs. 1 and 2).

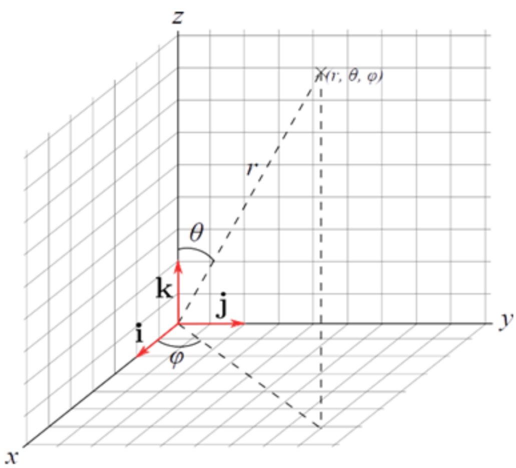

Fig. 1. Spherical coordinate system

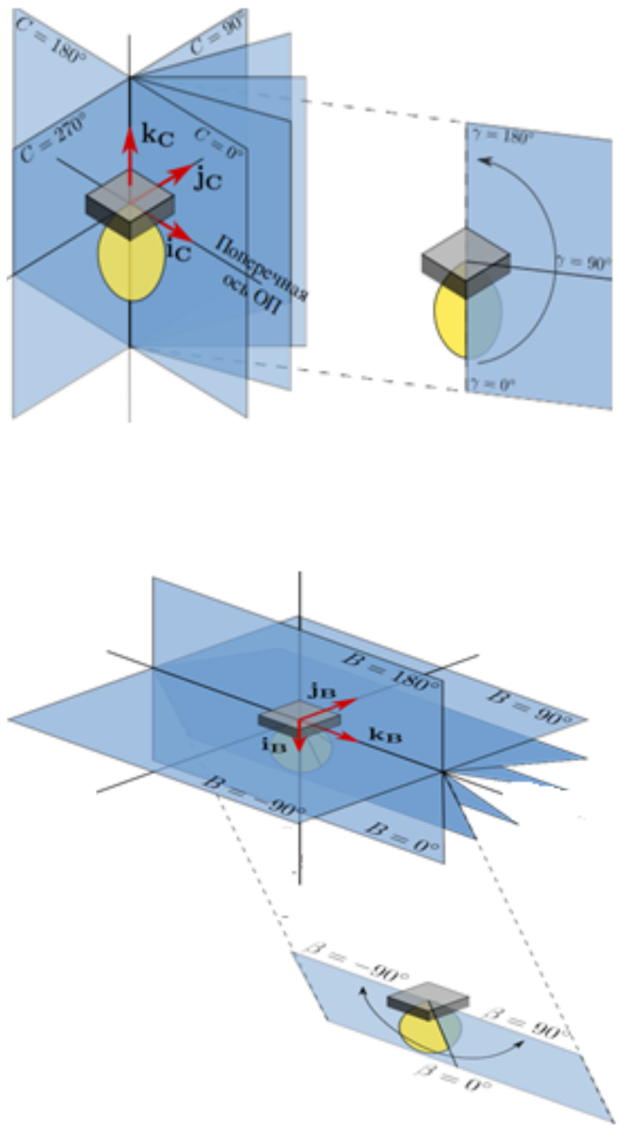

Fig. 2. Photometric system

Figures 1 and 2 show that the polar θ and azimuth φ angles are related to the meridian and equatorial angles of the systems Cγ, Bβ, Aα as follows: $\theta_{\mathrm{C}} = 180\,^{\circ} - \gamma$, $\varphi_{\mathrm{C}} = \mathrm{C}$, $\theta_{\mathrm{B}} = 90\,^{\circ} - \beta$, $\varphi_{\mathrm{B}} = \mathrm{B}$, $\theta_{\mathrm{A}} = 90\,^{\circ} + \alpha$, $\varphi_{\mathrm{A}} = \mathrm{A}$. Substituting these expressions in (1), we obtain the following formulas for the transformation of the system Cγ, Bβ, Aα coordinates in the rectangular coordinates

$$x_C = \sin\gamma\cos C,\ x_B = \cos\beta\cos B,\ x_A = \cos\alpha\cos A,$$

$$y_C = \sin\gamma\sin C,\ y_B = \cos\beta\sin B,\ y_A = \cos\alpha\sin A,$$

$$z_C = -\cos\gamma,\ z_B = \sin\beta,\ \_A = -\sin\alpha. \tag{2}$$

Now it is necessary to rotate the coordinate axes or the base of the systems. Turning again to Figs. 2–4 we see that during the transition C$\gamma \to$ Bβ, the rotation is 270 ° counterclockwise relative to the $\boldsymbol{j}_{\mathrm{C}}$ axis, and 270 ° counterclockwise for Bβ $\to$ Aα-relative to the $\boldsymbol{i}_{\mathrm{B}}$ axis. The matrices of the transformation data are given as

$$R_{\mathrm{cb}} = \begin{pmatrix} \cos 270^\circ & 0 & \sin 270^\circ \\ 0 & 1 & 0 \\ -\sin 270^\circ & 0 & \cos 270^\circ \end{pmatrix} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \tag{3}$$

$$R_{\mathrm{ba}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 270^\circ & -\sin 270^\circ \\ 0 & \sin 270^\circ & \cos 270^\circ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}. \tag{4}$$

As will be shown later, these two matrices are sufficient to describe fully the relationship of photometry systems. Now, let us give the mathematical notation for the transformations (4)

$$c = \begin{pmatrix} x_{\mathrm{C}} \\ y_{\mathrm{C}} \\ z_{\mathrm{C}} \end{pmatrix},\quad b = \begin{pmatrix} x_{\mathrm{B}} \\ y_{\mathrm{B}} \\ z_{\mathrm{B}} \end{pmatrix},\quad a = \begin{pmatrix} x_{\mathrm{A}} \\ y_{\mathrm{A}} \\ z_{\mathrm{A}} \end{pmatrix}.$$

The expressions (5)–10 determine the relationship between the photo-metering systems uniquely. Solving them in an explicit form, we find the following relations:

$$\mathrm{C}\gamma \to \mathrm{B}\beta:\ \mathrm{B} = \arctan(\sin\mathrm{C}\tan\gamma),\quad \beta = \arcsin(\cos\mathrm{C}\sin\gamma), \tag{5}$$

$$\mathrm{B}\beta \to \mathrm{A}\alpha:\ \mathrm{A} = \arctan(\tan\beta/\cos\beta),\quad \alpha = \arcsin(\sin\mathrm{B}\cos\beta), \tag{6}$$

$$\mathrm{C}\gamma \to \mathrm{A}\alpha:\ \mathrm{A} = \arctan(\cos\mathrm{C}\tan\gamma),\quad \alpha = \arcsin(\sin\mathrm{C}\sin\gamma), \tag{7}$$

$$\mathrm{B}\beta \to \mathrm{C}\gamma:\ \mathrm{C} = \arctan(\sin\beta/\tan\beta),\quad \gamma = \arccos(\cos\mathrm{B}\cos\beta), \tag{8}$$

$$\mathrm{A}\alpha \to \mathrm{B}\beta:\ \mathrm{B} = \arctan(\tan\alpha/\cos\mathrm{A}),\quad \beta = \arcsin(\sin\mathrm{A}\cos\alpha), \tag{9}$$

$$\mathrm{A}\alpha \to \mathrm{C}\gamma:\ \mathrm{C} = \arctan(\tan\alpha/\sin\mathrm{A}),\quad \gamma = \arccos(\cos\mathrm{A}\cos\alpha). \tag{10}$$

The same relations are given in [1]. Analyzing them and considering Figs. 1 and 2, we conclude that in order to find the equatorial angles A, B, and C, the use of only the main branch of the arc tangent is not enough, since $-\pi \leq \mathrm{A} \leq \pi$, $-\pi \leq \mathrm{B} \leq \pi$, and $0 \leq \mathrm{C} \leq 2\pi$, and at the same time $-\pi/2 < arctanx < \pi/2$. This leads to the fact that after the conversion, half the information is lost. For a correct determination of all values of the meridional angle, it should be looked for an argument of a complex number. Taking this into account, let us rewrite the expressions (5)–10) in the following form:

$$\mathrm{C}\gamma \to \mathrm{B}\beta: \ \mathrm{B} = \varphi(\sin\mathrm{C}, \cot\gamma), \quad \beta = \arcsin(\cos\mathrm{C}\sin\gamma), \tag{11}$$

$$\mathrm{B}\beta \to \mathrm{A}\alpha: \ \mathrm{A} = \varphi(\tan\beta, \cos\beta), \quad \alpha = \arcsin(\sin\mathrm{B}\cos\beta), \tag{12}$$

$$\mathrm{C}\gamma \to \mathrm{A}\alpha: \ \mathrm{A} = \varphi(\cos\mathrm{C}, \cot\gamma), \quad \alpha = \arcsin(\sin\mathrm{C}\sin\gamma), \tag{13}$$

$$\mathrm{B}\beta \to \mathrm{C}\gamma: \ \mathrm{C} = \varphi * (\sin\mathrm{B}, \tan\beta), \quad \gamma = \arccos(\cos\mathrm{B}\cos\beta), \tag{14}$$

$$\mathrm{A}\alpha \to \mathrm{B}\beta: \ \mathrm{B} = \varphi(\tan\alpha, / \cos\mathrm{A}), \quad \beta = \arcsin(\sin\mathrm{A}\cos\alpha), \tag{15}$$

$$\mathrm{A}\alpha \to \mathrm{C}\gamma: \ \mathrm{C} = \varphi * (\tan\alpha / \sin\mathrm{A}), \quad \gamma = \arccos(\cos\mathrm{A}\cos\alpha). \tag{16}$$

The result of the transformation by these formulas is shown in Fig. 3.

In general, the light intensity I is the function of two variables, the values of which are known only at the points (C_i, γ_j), but it is required to find out its value at the points from the other set $(\mathrm{C*}_k, \gamma*_l)$. To do this, we must perform the Delaunay triangulation of the domain $[0, 180\,°]\times[0, 360\,°]$. After this, the values of $I(\mathrm{C*}_k, \gamma*_l)$ can be obtained by the interpolation of $I(\mathrm{C}_i, \gamma_j)$ values, at the vertices of the triangles. The problem is that the triangulation grid constructed from the available (C_i, γ_j) covers the domain of the function I definition not completely: the edges of the domain do not fall into any triangle (Fig. 6). This leads to the impossibility of interpolation at some points of the set $(\mathrm{C*}_k, \gamma*_l)$; the photometric body itself will have a "cutout". This difficulty can be overcome by using the parity and the periadicity of LDC, which follow from the definition of the spherical coordinate:

$$I(-C,\gamma) = I(C,\gamma), \ I(360° - C,\gamma) = I(C,\gamma)\,,$$

$$I(-C,\gamma - 360°) = I(C,\gamma), \ I(C,\gamma + 360°) = I(C,\gamma)\,,$$

These relations make it possible to determine (C,γ) at the points lying beyond the lower, the upper, the left and the right boundaries of its definition domain, respectively. Expanding these boundaries, it is possible to ensure that the original rectangle $[0°, 180°]\times[0°, 360°]$, together with all the points $(\mathrm{C*}_k, \gamma*_l)$, will be covered by Delaunay triangulation grid completely, built for a new expanded domain of definition.

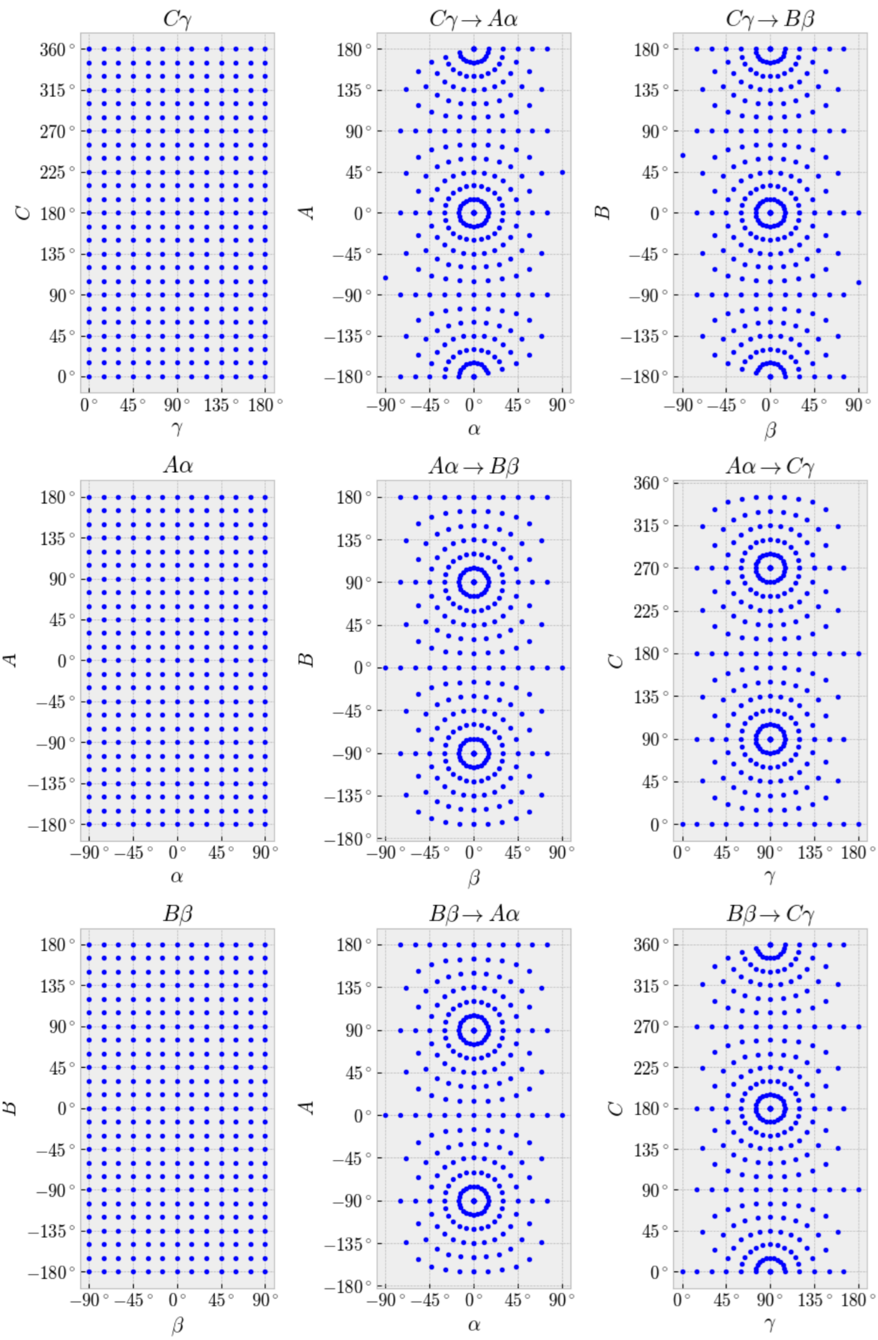

Fig. 3. Photometric system nodes before and after conversion by formulas 11–16

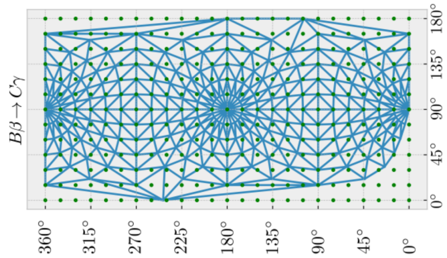

Fig. 4. Interpolation by scattered data

3. Conclusion

The paper showed that, after the transformations (11–16), the grid of angles becomes irregular, which in its turn leads to the inability to generate photometric data files in widely distributed formats ldt and ies. It is shown that in order to solve the problem of a regular grid of angles, one can use the interpolation of scattered data on the basis of Delaunay triangulation. The solution for photometric system conversion proposed by the authors is more universal. Secondly, [1, 3] do not indicate that after the transformation B$\beta \to$ Cγ the grid of angles becomes irregular one, with all the ensuing consequences. And, thirdly, in order to transform A$\alpha \to$ Cγ the formulas are proposed in [3] that do not agree with either [1] or with the solution proposed in this paper.

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Received October 12, 2017